

# BPS Solutions to a Generalized Maxwell-Higgs Model

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We look for topological BPS solutions of an Abelian-Maxwell-Higgs theory endowed by non-standard kinetic terms to both gauge and scalar fields. Here, the non-usual dynamics are controlled by two positive functions,  $G(|\phi|)$  and  $w(|\phi|)$ , which are related to the self-dual scalar potential  $V(|\phi|)$  of the model by a fundamental constraint. The numerical results we found present interesting new features, and contribute to the development of the recent issue concerning the study of generalized models and their applications.

PACS numbers: 11.10.Kk, 11.10.Lm

## I. INTRODUCTION

In the context of classical field theories, topological structures, such as kinks [1], vortices [2] and magnetic monopoles [3], are described as static solutions to some nonlinear models. In particular, such models must allow for the spontaneous symmetry breaking mechanism, since topological solutions are formed during a symmetry breaking phase transition. In this sense, topologically non-trivial configurations are of great interest to physics [4], mainly concerning the cosmological consequences they may engender, since such configurations can appear in a rather natural way, during phase transitions in the early universe.

In particular, vortices are described as rotationally symmetric solutions of a planar Abelian-Maxwell-Higgs model endowed by a fourth-order scalar potential for the matter self-interaction, which also introduces symmetry breaking and nonlinearity. In the usual case, the Maxwell term controls the dynamics of the gauge field, and the covariant derivative squared term controls the dynamics of the scalar field. In such context, vortices are finite-energy solutions of a set of two coupled first-order differential equations, named Bogomol'nyi-Prasad-Sommerfield (BPS) equations [5]. In this case, the BPS vortices are the minimal-energy solutions of the model, and they have interesting applications, mainly concerning the superconductivity phenomena [2] and the superfluid  $\text{He}^4$ .

During the last years, beyond the standard configurations previously cited, modified ones, also named *topological k-solutions*, have been intensively studied. These solutions arise in a special class of theoretical field models, generically named *k-theories*, which are endowed by non-usual kinetic terms. As expected, such terms change the dynamics of the overall model under investigation. Moreover, it is important to reinforce that the idea of a non-standard dynamics arises in a rather natural way, in the context of string theories.

In general, k-theories have been used as effective models mainly in the cosmological scenario. Here, many authors have used the so-called k-essence models [6] to investigate the present accelerated inflationary phase of the universe [7]. Also, such models can be used to study strong gravitational waves [8], dark matter [9], tachyon matter [10] and others [11]. There are also important motivations concerning the strong interaction physics; see, for instance, Ref. [12].

In this context, the overall conclusion is that the existence of topological structures is quite sensible to the use of non-standard kinetic terms. Here, the more interesting issue is that such structures can exist even as solutions of some theories which do not allow for the spontaneous symmetry breaking mechanism [13]. On the other hand, a rather natural way to study such topological solutions is comparing them with their canonical counterparts. In this sense, some of us have already investigated topological solutions in the context of k-field models endowed by spontaneous symmetry breaking potentials for the scalar-matter self-interaction, and interesting results can be found, for instance, in Ref. [14]. Other interesting results concerning k-field models and their classical solutions can be found, for instance, in Ref. [15].

Another interesting issue concerning topological k-solutions is that they can be either much larger or much smaller than their usual counterparts. In this sense, k-bosons can mediate either large-range or small-range interactions. Also, important physical quantities, such as energy density and electric and magnetic fields, can exhibit prominent variations on their typical profiles, including on their maximum values; for a detailed treatment of some of such features, see Ref. [16].

In the present context, some k-theories can also support topological solutions with a finite wavelength. These solutions, generically named *compactons* [17], are quite different from the standard topological structures, which interact even if separated by an infinite displacement, since they have an infinite wavelength: two adjacent com-

pactons will interact only if they come into close contact, due to their already explained finite wavelength. In this sense, compactons are most appropriated to describe particle-like configurations than the usual non-compactified topological structures.

So, in this work we present new results concerning the topological solutions of an Abelian-Maxwell-Higgs model endowed by non-usual kinetic terms to both gauge and scalar fields. Here, the non-standard dynamics are introduced by two positive functions,  $G(|\phi|)$  and  $w(|\phi|)$ , which couple with the Maxwell term and with the covariant derivative squared term, respectively. The Euler-Lagrange equations of motion of such theory are hard to solve, then we focus our attention only on the finite-energy solutions of the BPS equations of the model. These equations can be obtained by minimizing the energy functional of the model, which can be done via an important constraint between  $G(|\phi|)$ ,  $w(|\phi|)$  and the scalar potential  $V(|\phi|)$  for the matter self-interaction; see eqs.(15) and (16) below. In this case, rotationally symmetric BPS vortices are described as the minimal-energy solutions of the cited model, and they engender interesting features, as explained below.

This paper is outlined as follows: in the next Sec.II, we introduce the model and develop the theoretical framework which allows us to get to its Bogomol'nyi-Prasad-Sommerfield equations. In Sec.III, we prove the consistence of the theoretical framework previously developed by using it to investigate the existence of new BPS states. Here, we note that such states are constrained to the choices made for  $G(|\phi|)$  and  $w(|\phi|)$ : for any acceptable pair of such functions, there is a corresponding BPS configuration. Also in Sec.III, we show how to use the theoretical framework presented in this work to recover the recover the BPS results concerning the standard Maxwell-Higgs model. In Section IV, we perform the numerical analysis concerning the new BPS states previously presented, we depict the corresponding minimal-energy modified solutions and comment on their main features. Finally, in Sec.V, we present our conclusions and perspectives.

From now on, we use standard conventions, including natural units system, and a plus-minus signature for the planar Minkowski metric:  $\text{diag}(\eta^{\mu\nu}) = (+ - -)$ .

## II. THE MODEL

In this section, we introduce the model. It is described by the  $(2+1)$ -dimensional Lagrange density

$$\mathcal{L}_G = -\frac{1}{4}G(|\phi|)F_{\mu\nu}F^{\mu\nu} + w(|\phi|)|D_\mu\phi|^2 - V(|\phi|) . \quad (1)$$

Here,  $F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$  is the usual Faraday field strength tensor,  $D_\mu\phi = \partial_\mu\phi + ieA_\mu\phi$  is the covariant derivative and  $V(|\phi|)$  is the spontaneous symmetry breaking potential. Also,  $G(|\phi|)$  is the "dielectric function", and  $w(|\phi|)|D_\mu\phi|^2$  stands for the non-standard ki-

netic matter term. Both  $G(|\phi|)$  and  $w(|\phi|)$  are dimensionless functions to be given below, as functions of the amplitude of the scalar field.

In the case of vortices, it is convenient to deal with dimensionless variables. So, for simplicity, we introduce the mass scale  $M$ , and use it to implement the scale transformations:  $x^\mu \rightarrow x^\mu/M$ ,  $\phi \rightarrow M^{1/2}\phi$ ,  $A^\mu \rightarrow M^{1/2}A^\mu$ ,  $e \rightarrow M^{1/2}e$  and  $v \rightarrow M^{1/2}v$ , where  $v$  stands for the symmetry breaking parameter of the model. In this case, we get  $\mathcal{L}_G \rightarrow M^3\mathcal{L}$ , with  $\mathcal{L}$  being the dimensionless Lagrange density to be used from now on, which has the same functional form of  $\mathcal{L}_G$ . Moreover, we omit the coupling constant related to the scalar matter self-interaction. Also, we take  $e$  and  $v$  as real and positive parameters.

To search for vortex solutions, the canonical procedure is to deal with the Euler-Lagrange equations of the model. In the present case, these equations are

$$G\partial_\mu F^{\mu\lambda} + F^{\mu\lambda}\partial_\mu G = J^\lambda , \quad (2)$$

and

$$\begin{aligned} & w\partial_\mu\partial^\mu|\phi| + \partial_\mu|\phi|\partial^\mu w - e^2 A_\mu A^\mu |\phi| w \\ &= \frac{1}{2}|D_\mu\phi|^2 \frac{dw}{d|\phi|} - \frac{F^2}{8} \frac{dG}{d|\phi|} - \frac{1}{2} \frac{dV}{d|\phi|} . \end{aligned} \quad (3)$$

Here, we take  $F^2 = F_{\mu\nu}F^{\mu\nu}$  and

$$J^\mu = -2e^2 w |\phi|^2 A^\mu \quad (4)$$

as the modified 4-current vector.

The Gauss law for time-independent fields can be written as

$$G\partial_k\partial^k A^0 + \partial_k A^0\partial^k G = -2e^2 w |\phi|^2 A^0 , \quad (5)$$

where  $\mathbf{E} = -\vec{\nabla}A^0$ . We note that the Eq. (5) is trivially verified by  $A^0 = 0$ . So, we fix this gauge and use it from now on.

We look for vortex solutions of the form

$$\phi(r, \theta) = v g(r) e^{in\theta} , \quad (6)$$

$$\mathbf{A} = -\frac{\hat{\theta}}{er} (a(r) - n) . \quad (7)$$

Here,  $r$  and  $\theta$  are polar coordinates, and  $n = \pm 1, \pm 2, \pm 3, \dots$  is the winding number (vorticity) of the solution. In terms of (6) and (7), the Euler-Lagrange equations (2) and (3) can be rewritten as

$$G \frac{d^2 a}{dr^2} + \left( \frac{dG}{dr} - \frac{G}{r} \right) \frac{da}{dr} = 2e^2 v^2 g^2 a w , \quad (8)$$

$$\begin{aligned} & w \left( \frac{d^2 g}{dr^2} + \frac{1}{r} \frac{dg}{dr} - \frac{a^2 g}{r^2} \right) - \frac{1}{4v^2} \left( \frac{1}{er} \frac{da}{dr} \right)^2 \frac{dG}{dg} \\ &= \frac{1}{2v^2} \frac{dV}{dg} - \frac{1}{2} \left( \left( \frac{dg}{dr} \right)^2 - \frac{g^2 a^2}{r^2} \right) \frac{dw}{dg} . \end{aligned} \quad (9)$$

Equations (8) and (9) are the Euler-Lagrange equations of motion to the profile functions  $a(r)$  and  $g(r)$ , respectively.

To solve (8) and (9), we need to specify the model. In general, we can do it by choosing non-trivial forms to  $G(g)$  and  $w(g)$ . In this case, we have to keep in mind that both these functions must be positive, in order to avoid problems with the energy of the model; see the expression for the energy density (15) below. Also, we need to choose a Higgs potential  $V(g)$  which allows for the spontaneous symmetry breaking mechanism.

Before that, we note that the limit  $w(g) \rightarrow 1$  leads us back to the model studied in [18], which is supported by applications concerning the interaction between quarks and gluons [19]. In this case, the limit  $G(g) \rightarrow 1$  leads us back to the usual Maxwell-Higgs theory. In this context, if we choose the symmetry breaking Higgs potential

$$V_s(g) = \frac{e^2 v^4}{2} (1 - g^2)^2, \quad (10)$$

the Euler-Lagrange equations (8) and (9) can then be rewritten as

$$\frac{d^2 a}{dr^2} - \frac{1}{r} \frac{da}{dr} = 2e^2 v^2 g^2 a, \quad (11)$$

$$\frac{d^2 g}{dr^2} + \frac{1}{r} \frac{dg}{dr} - \frac{a^2 g}{r^2} = e^2 v^2 g (g^2 - 1). \quad (12)$$

According to our conventions, eqs.(11) and (12) are completely solvable by the first order differential equations

$$\frac{dg}{dr} = \pm \frac{ga}{r}, \quad (13)$$

$$\frac{1}{r} \frac{da}{dr} = \pm e^2 v^2 (g^2 - 1). \quad (14)$$

The solutions of eqs.(13) and (14) are the Bogomol'nyi-Prasad-Sommerfield (BPS) states of the standard Maxwell-Higgs model (10). These solutions are the well-known Abrikosov-Nielsen-Olesen (ANO) ones [2], which solve the equations of motion (11) and (12) by minimizing the energy of the resulting vortex configurations.

In general, for non-trivial choices to  $G(g)$  and  $w(g)$ , the equations of motion (8) and (9) will be much more sophisticated than the eqs.(11) and (12). So, in order to get an useful insight about the non-trivial case, we consider the expression for the energy density of the modified vortex solutions:

$$\varepsilon = \frac{G}{2e^2} \left( \frac{1}{r} \frac{da}{dr} \right)^2 + v^2 w \left( \left( \frac{dg}{dr} \right)^2 + \frac{g^2 a^2}{r^2} \right) + V. \quad (15)$$

From Eq. (15), we note that the presence of non-trivial  $G(g)$  and  $w(g)$  makes it hard to obtain the BPS states of the modified model. Even in this case, the existence

of such states is still possible, and it is closely related to an important constraint between  $G(g)$ ,  $w(g)$  and  $V(g)$ :

$$\frac{d}{dg} \sqrt{GV} = \sqrt{2} e v^2 w g. \quad (16)$$

As we clarify below, for any positive choices to  $G(g)$  and  $w(g)$ , there is a corresponding symmetry breaking Higgs potential  $V(g)$  as a solution of (16). In this context, generalized first order equations can be found by minimizing the energy functional (15).

To search for BPS vortex solutions in the modified model, we need to know how the functions  $g(r)$  and  $a(r)$  behave, near the origin and asymptotically. Near the origin, these functions must avoid singular fields. So, they have to behave according to

$$g(r \rightarrow 0) \rightarrow 0 \quad \text{and} \quad a(r \rightarrow 0) \rightarrow n = 1, \quad (17)$$

where we have fixed  $n = 1$ , for simplicity. Also, the symmetry breaking vortex configurations must have a finite total energy. So, as a condition to make the energy finite, the energy density (15) must vanish. Then, asymptotically,  $g(r)$  and  $a(r)$  have to obey

$$g(r \rightarrow \infty) \rightarrow 1 \quad \text{and} \quad a(r \rightarrow \infty) \rightarrow 0. \quad (18)$$

In the next Sec. III, we use the theoretical framework developed in this Section to investigate the existence of BPS states in the generalized model. Also, we present and comment the resulting numerical solutions. Finally, we show how to map the standard Maxwell-Higgs and Chern-Simons-Higgs first order equations; even in these cases, Eq. (16) leads us to a physically different rotationally symmetric solutions.

### III. NEW BPS STATES

We now pay due attention to the modified BPS states themselves. Here, we choose positive functional forms to  $G(g)$  and  $w(g)$ . Then, we solve the constraint (16) to get to the consistent Higgs potential  $V(g)$ , which must allow for the spontaneous symmetry breaking mechanism. A posteriori, we use these conventions to obtain the first order differential equations of the modified model, by minimizing its energy functional (15). Then, in the next Sec.IV, we numerically solve these equations, according to the boundary conditions (17) and (18). Finally, in Sec.V, we comment the main features of the resulting solutions.

The model to be studied here has two unusual functions,  $G(g)$  and  $w(g)$ , which we have to fix to determine how they affect the vortex solutions. The function  $G(g)$  stands for a "dielectric function", and it controls the non-standard kinetic term to the gauge field. On the other hand, the function  $w(g)$  controls the non-usual kinetic scalar matter term. Both these functions are dimensionless, and are functions of the amplitude of the scalar field.

Also, they must be positive, in order to avoid problems with the energy of the model; see Eq. (15).

A rather natural way to investigate the consistence of the theoretical framework developed in the previous Sec. II is to study the BPS states of the standard Maxwell-Higgs model, which is defined by  $G(g) = w(g) = 1$ . In this case, Eq. (16) leads us to the potential (10). Then, the energy density (15) can be rewritten as

$$\begin{aligned} \varepsilon = & \frac{1}{2} \left( \frac{1}{er} \frac{da}{dr} \mp ev^2 (g^2 - 1) \right)^2 + v^2 \left( \frac{dg}{dr} \mp \frac{ga}{r} \right)^2 \\ & \mp \frac{v^2}{r} \frac{da}{dr} \pm \frac{v^2}{r} \frac{d}{dr} (g^2 a) . \end{aligned} \quad (19)$$

The resulting total energy  $E$  is minimized by the first order equations (13) and (14). In this context, the energy of the BPS states is

$$E_s = \int \varepsilon(r) d^2r = 2\pi v^2 |n| . \quad (20)$$

As usual, this is the lower bound of the energy functional, i.e., the Bogomol'nyi bound. Also, it is quantized according to the winding number  $n$ .

Now, we introduce an interesting new model, which is defined by

$$G(g) = \frac{(g^2 + 3)^2}{g^2} \quad \text{and} \quad w(g) = 2(g^2 + 1) . \quad (21)$$

According to these choices, we solve (16) to get to the potential

$$V(g) = g^2 V_s(g) , \quad (22)$$

which allows for the spontaneous symmetry breaking mechanism, as desired. We point out that this new model is not a parametrization of the standard Maxwell-Higgs one, since the vacuum manifold of the two models are quite different: it is a circle for the usual model (10), while it is a dot surrounded by a circle for the new model (22).

Using eqs.(21) and (22), the energy density (15) can then be rewritten as

$$\begin{aligned} \varepsilon = & \frac{(g^2 + 3)^2}{2g^2} \left( \frac{1}{er} \frac{da}{dr} \mp \frac{ev^2 g^2 (g^2 - 1)}{(g^2 + 3)} \right)^2 \\ & + 2v^2 (g^2 + 1) \left( \frac{dg}{dr} \mp \frac{ga}{r} \right)^2 \\ & \pm \frac{v^2}{r} \frac{d}{dr} (a (g^2 - 1) (g^2 + 3)) . \end{aligned} \quad (23)$$

Then, the corresponding energy functional is minimized by the equations

$$\frac{dg}{dr} = \pm \frac{ga}{r} , \quad (24)$$

$$\frac{1}{r} \frac{da}{dr} = \pm \frac{e^2 v^2 g^2 (g^2 - 1)}{(g^2 + 3)} . \quad (25)$$

We note that the modified equations (24) and (25) can not be parametrized into the standard ones, i.e., eqs.(13) and (14). Even in this case, the energy of the modified field solutions is bounded from below, and the Bogomol'nyi bound is

$$E = 3E_s . \quad (26)$$

Here, as in the standard case, the total energy of the BPS states is quantized; see Eq. (20).

In order to reinforce the consistence of the theoretical framework presented in this work, we introduce another modified model. It is defined by

$$G(g) = (g^2 + 1)^2 \quad \text{and} \quad w(g) = 2g^2 . \quad (27)$$

According to these conventions, Eq. (16) leads us to the standard Higgs potential (10). So, the modified model (27) has the same vacuum structure as the usual model. Even in this context, the new model is not a parametrization of the standard one; see the BPS eqs.(29) and (30) below. The modified energy density can be written in the form

$$\begin{aligned} \varepsilon = & \frac{(g^2 + 1)^2}{2} \left( \frac{1}{er} \frac{da}{dr} \mp \frac{ev^2 (g^2 - 1)}{(g^2 + 1)} \right)^2 \\ & + 2v^2 g^2 \left( \frac{dg}{dr} \mp \frac{ga}{r} \right)^2 \\ & \pm \frac{v^2}{r} \frac{d}{dr} (a (g^2 - 1) (g^2 + 1)) , \end{aligned} \quad (28)$$

and the resulting energy functional is minimized by the first order equations

$$\frac{dg}{dr} = \pm \frac{ga}{r} , \quad (29)$$

$$\frac{1}{r} \frac{da}{dr} = \pm \frac{e^2 v^2 (g^2 - 1)}{(g^2 + 1)} . \quad (30)$$

Then, the Bogomol'nyi bound is also given by (20), and we conclude that the modified solutions (29) and (30) have the same energy of the standard (13) and (14) ones.

To end this Section, we show how to use the theoretical framework developed in this work to map the standard Maxwell- and Chern-Simons-Higgs first order differential equations. We do it by reviewing the model studied in [18], which is defined by  $w(g) = 1$ . In this case, as presented in that work, an interesting choice to the dielectric function is

$$G(g) = (1 + \lambda) \left( 1 + 2\lambda \frac{e^2 v^2}{k^2} g^2 \right)^{-1} . \quad (31)$$

Here,  $\lambda$  is a real auxiliary parameter which controls the model, and  $k$  stands for the coupling constant corresponding to the Chern-Simons term. In the limit  $\lambda \rightarrow 0$ ,

Eq. (31) gives  $G(g) \rightarrow 1$ , and (16) reproduces the standard Maxwell-Higgs system (10). On the other hand, the limit  $\lambda \rightarrow \infty$  gives

$$G(g) = \frac{k^2}{2e^2v^2g^2} . \quad (32)$$

We then solve Eq. (16) to get

$$V(g) = \frac{e^4v^6}{k^2}g^2(g^2-1)^2 , \quad (33)$$

which is exactly the sixth-order symmetry breaking Higgs potential one finds in the usual self-dual Chern-Simons-Higgs model. Now, from eqs.(32) and (33), the energy density (15) can be written in the form

$$\begin{aligned} \varepsilon = & \frac{k^2}{4e^2v^2g^2} \left( \frac{1}{er} \frac{da}{dr} \mp \frac{2e^3v^4}{k^2} g^2 (g^2-1) \right)^2 \\ & + v^2 \left( \frac{dg}{dr} \mp \frac{ga}{r} \right)^2 \pm v^2 \frac{1}{r} \frac{d}{dr} (a(g^2-1)) . \end{aligned} \quad (34)$$

The resulting total energy is minimized by the differential equations

$$\frac{dg}{dr} = \pm \frac{ga}{r} , \quad (35)$$

$$\frac{1}{r} \frac{da}{dr} = \pm \frac{2e^4v^4}{k^2} g^2 (g^2-1) , \quad (36)$$

and the Bogomol'nyi bound is  $E = E_s$ . The equations (35) and (36) are just the BPS ones for the standard Chern-Simons-Higgs system (33). In this sense, Eq. (31) works as a unified way to map both the Maxwell- and the Chern-Simons-Higgs first order differential equations. Finally, we point out that, despite the possibility of reproducing such Chern-Simons equations, the unified picture (31) leads us to a physically different situation, since the modified model we consider here only supports non-charged field solutions; see the Gauss law (5) above. In this case, the modified nontopological field solutions are unstable, and they can decay into the elementary mesons of the model. This fact makes an important difference, since the nontopological configurations presented by the self-dual Chern-Simons-Higgs theory are stable, and can not decay.

In the next Section, we present the modified first order numerical solutions for the profile functions  $g(r)$  and  $a(r)$ . Also, we comment on the main features of such solutions.

#### IV. NUMERICAL SOLUTIONS

Let us focus attention on the modified numerical solutions themselves. The equations to be solved are the first order ones (24) and (25), and (29) and (30). Also, we solve the standard equations (13) and (14), for comparison. In all cases, the functions  $g(r)$  and  $a(r)$  must obey the boundary conditions (17) and (18).

In the present work, the numerical strategy to be employed is the relaxation one. In this case, it is necessary to input an approximated field solution. Then, our algorithm will "relax" it into the correct one. To start the numerical analysis, we consider a variation of the standard Maxwell-Higgs model, in which the potential for the self-interaction for the scalar field is not present. Then, we use its solutions to solve the self-dual Maxwell-Higgs case, i.e., the equations (13) and (14), from which we get the well-understood Abrikosov-Nielsen-Olesen (ANO) configurations. We then use these solutions to initialize the numerical study of the modified theory.

Starting from such ANO configurations, we numerically solve the modified BPS equations presented in the previous Section, i.e., eqs.(24) and (25), and also eqs.(29) and (30), for  $e = v = n = 1$ . The solutions for the profile functions  $g(r)$  and  $a(r)$  are plotted in Figs. 1 and 2, respectively. Also, we depict the solutions for the corresponding energy densities; see Figure 3.

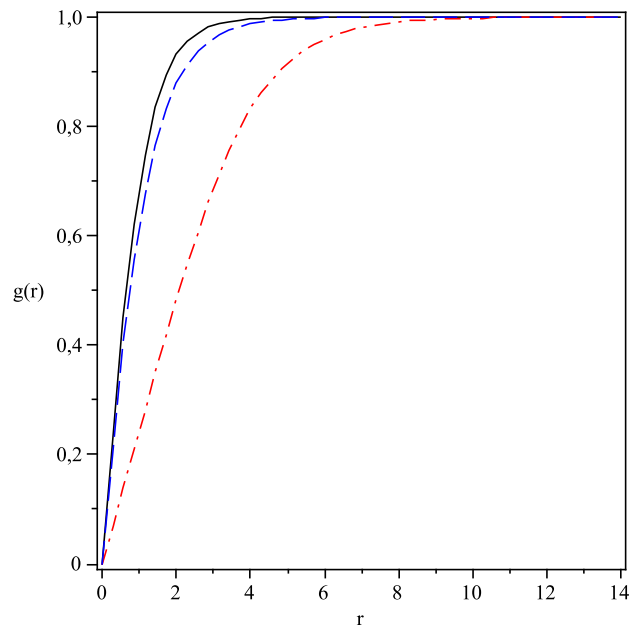


FIG. 1: Numerical BPS solutions to  $g(r)$  for the models (21) (dash-dotted red line) and (27) (dashed blue line). The standard solution is also depicted (solid black line), for comparison.

In Figure 1, we present the solutions for the profile function  $g(r)$ , and we see that both the modified profiles reach their vacuum values more slowly than their canonical counterpart. In this sense, such solutions have a core which is greater than the usual Maxwell-Higgs one. Here, the conclusion is that, in general, the introduction of a non-canonical dynamics allows for the existence of self-dual field solutions  $g(r)$  with a increased characteristic length. Also, we note that the solution for (24) and (25) goes to its vacuum configuration more slowly than that for (29) and (30). So, beyond the fact of increasing the core of the solutions, we note that (21) increases it more

than the (27). We believe that this fact is related with the corresponding self-dual Higgs potential, which is of sixth-order for the model (21), and of fourth-order for the (27) one; see equations (22) and (10) above.

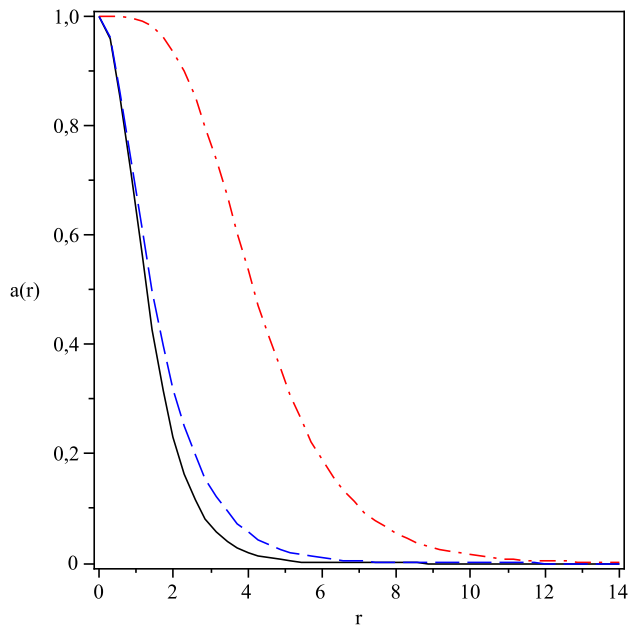


FIG. 2: Numerical BPS solutions to  $a(r)$ . Conventions as in FIG. 1.

In Fig. 2, we depict the numerical solutions for the function  $a(r)$ . Here, as in the Fig. 1, we note that the modified solutions go to their vacuum states more slowly than the standard Maxwell-Higgs profile, and so such solutions have a increased characteristic length. This behaviour reinforces the previous conclusion, which states that a non-standard dynamics leads to a self-dual field solutions with a increased core. We also see that, beyond the fact of increasing the core of  $a(r)$ , the model (21) enlarges it more than the (27). Finally, we point out the existence of a prominent plateau in the profile related to (21), near the origin: such structure is also present in the self-dual Chern-Simons-Higgs case, which is governed by the potential (33). In this context, it is interesting to note that the existence of such prominent plateau seems to be closely related to the vacuum manifold of a sixth-order symmetry breaking potential, since it is also present in the modified model (21), which is governed by (22).

The Fig. 3 encloses the solutions for the energy densities of the new BPS states; see equations (23) and (28). We see that the profile for the energy density corresponding to the modified model (21) is quite different from that of the standard Maxwell-Higgs theory: in the canonical case, the energy density reaches its maximum value in the limit  $r \rightarrow 0$ , and it is monotonically decrescent for all  $r$ . On the other hand, in the modified case (21), the energy density reaches its maximum value at some finite distance  $R$  from the origin, and it is not monotonically

decrecent for all values of the independent variable. In this context, this difference reinforces our previous conclusion, since such modified behaviour mimics the one of the energy density of the usual self-dual Chern-Simons theory.

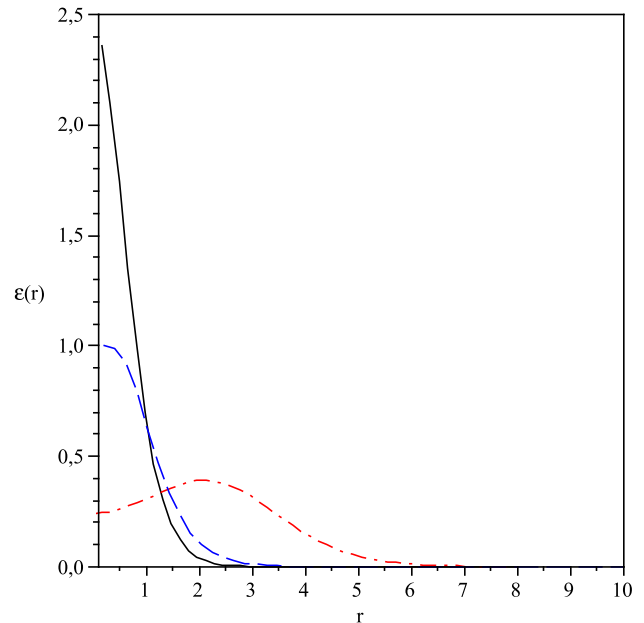


FIG. 3: Plots of the energy density  $\varepsilon(r)$ . Conventions as in FIG. 1.

The solution for the BPS energy density of the modified model (27) is also depicted in Figure 3, and we see that its behaviour is qualitatively similar to that of the Maxwell-Higgs model: it reaches its maximum value as  $r$  goes to 0, and it is monotonically decrescent for all  $r$ . Here, however, there are two important differences: the first one is on the maximum value of the modified profile itself, which is smaller than the usual one, and the second one is on the characteristic length of the non-standard solution, which is greater than its canonical counterpart.

An important issue concerning the study of topological structures is the computation of their topological charges, which must be conserved. In the present case, i.e., for electrically non-charged field solutions of the form (6) and (7), this charge is related to the magnetic field generated by such solutions themselves. To investigate this issue, we introduce the topological current

$$J^\mu = \epsilon^{\mu\nu\lambda} \partial_\nu A_\lambda, \quad (37)$$

which is clearly conserved. Here,  $\epsilon^{012} = -1$ . The 0<sup>th</sup>-component of such current, that is, the topological charge density, can be written as

$$J^0 = \partial_x A_y - \partial_y A_x = B, \quad (38)$$

and we see that it is directly proportional to the magnetic field  $B$ . In this case, the topological charge is given by

$$Q_T = \int B d^2r \equiv \Phi_B. \quad (39)$$

Here,  $\Phi_B$  stands for the flux of the magnetic field.

According (7), (17) and (18), the magnetic flux  $\Phi_B$  can be rewritten in the form

$$\Phi_B = \frac{2\pi n}{e}, \quad (40)$$

and we note that the topological charge  $Q_T$  is conserved, and it is quantized according to the winding number  $n$ .

In this sense, to better specify the field configurations studied in this work, a rather natural way is to consider their corresponding magnetic fields. In the present context, this field is given by

$$B = -\frac{1}{r} \frac{da}{dr}. \quad (41)$$

The modified numerical profiles for the magnetic field (41) are plotted in Figure 4. Also for such field, the non-usual dynamics introduced earlier leads to BPS solutions with a increased core, since the modified profiles have a characteristic length which is greater than the standard one. In this case, as for the energy densities previously depicted, the magnetic field associated to (21) is quite different from the canonical self-dual one, which reaches its maximum value for  $r \rightarrow 0$ , and is monotonically decrescent ever: the non-standard solution reaches its maximum value at some finite distance from the origin, and it is not monotonically decrescent for all  $r$ . Such behaviour is similar to that of the self-dual magnetic field related to the canonical Chern-Simons-Higgs model.

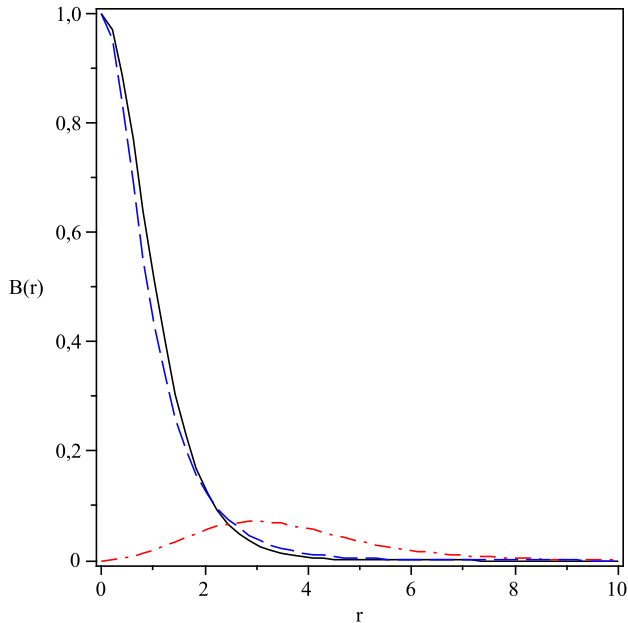


FIG. 4: Plots of the magnetic field  $B(r)$ . Conventions as in FIG. 1.

To end this Section, we discuss the solution for the magnetic field (41) related to the model (27). This solution is also depicted in Fig. 4, from which we note that the behaviour of such magnetic field is quite similar to

that of the usual Maxwell-Higgs case, since it reaches its maximum value as  $r \rightarrow 0$ , and it is monotonically decrescent ever. Here, we point out the behaviour of the modified solution, which assures the conservation of the topological charge; see eqs.(39) and (40).

## V. ENDING COMMENTS

In the present letter, we have considered the existence of rotationally symmetric BPS solutions in a  $(2+1)$ -dimensional space-time. We have investigated such solutions in a modified self-dual Maxwell-Higgs model endowed by a non-standard dynamics. Here, the modification was introduced in terms of two non-trivial functions,  $G(|\phi|)$  and  $w(|\phi|)$ , which must be positive, in order to avoid problems with the energy density (15) of the model. So, while  $G(|\phi|)$  couples with the Maxwell term and, as a consequence, changes the dynamics of the gauge field in a non-usual way,  $w(|\phi|)$  couples with the squared covariant derivative of the non-charged scalar field, then leading to a non-standard dynamics to such field. In this context, consistent first order equations were obtained since the non-trivial functions  $G(|\phi|)$  and  $w(|\phi|)$  are constrained to the symmetry breaking Higgs potential  $V(|\phi|)$  of the modified model; see Eq. (16).

We have integrated the BPS equations by means of the relaxation method, and the numerical results we found are depicted in Figs. 1 and 2, for some interesting choices to the functions  $G(|\phi|)$  and  $w(|\phi|)$ ; see eqs.(21) and (27). According these solutions, we conclude that both the profile functions  $g(r)$  and  $a(r)$  are quite sensible to the choices made for  $G(|\phi|)$  and  $w(|\phi|)$ . In particular, it is important to reinforce that each of the models (21) and (27) is related to a very specific symmetry breaking potential  $V(|\phi|)$  and, as a consequence, such models present distinct vacuum manifolds; for details, see eqs.(10) and (22).

Also, using the previous results we found for  $g(r)$  and  $a(r)$ , we have integrated both the energy density  $\varepsilon(r)$  and the magnetic field  $B(r)$  for the modified models (21) and (27), and these solutions are depicted in Figs. 3 and 4, respectively; see eqs.(23), (28) and (41). The numerical analysis reveals that the solutions corresponding the model (21) behave as those predicted by the Chern-Simons-Higgs theory, which reach their maximum values at some finite distance from the origin, and are not monotonically decrescent for all  $r$ . On the other hand, the solutions corresponding the model (27) behave as the Maxwell-Higgs ones, since they reach their maximum values as  $r \rightarrow 0$ , and are monotonically decrescent for all values of the independent variable. So, as a consequence, we conclude that also the energy density  $\varepsilon(r)$  and the magnetic field  $B(r)$  are both sensible to the choices made for  $G(|\phi|)$  and  $w(|\phi|)$ .

We hope that this work may stimulate subsequent analysis in the field, concerning mainly the features that the modified solutions engender. Also, we point out that

the variation on the characteristic length of the modified solutions presented here is closely related to the effects of anisotropy, which is a feature typically present in the effective field models used to describe the behaviour engendered by low-energy condensed matter systems. In this sense, we point out that such effective models are usually based on the Lorentz-breaking idea, since it introduces the issue of anisotropy explicitly. In the context of Lorentz-violating models, such effects were already studied by some of us; see, for instance, Ref. [20].

A very interesting issue concerns the use of modified

models to mimic the very same solutions engendered by the standard Maxwell-Higgs theory. In particular, such issue is now under consideration, and we hope to report new results in a near future.

The authors would like to thank CAPES, CNPq (Brazil) and FCT Project CERN/FP/116358/2010 (Portugal) for partial financial support. Also, we are grateful to C. Adam, F. Correia and D. Rubiera-Garcia for useful discussions. E. da Hora thanks the Department of Mathematical Sciences of Durham University (United Kingdom), for all their hospitality while doing this work.

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